Final-State Interactions in $B \to (\pi\pi)_{S,P} K$ and $B \to (\bar{K}K)_{S,P} K$ Decays: f_0 and ρ Resonances

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Motivation

- Experimental results at SLAC and KEK (charmless three-body decays more frequent than two-body ones)
- Direct CP-violation
- Establish the role of hadronic long-distance effects (*charming penguins, meson-meson interactions*)
- Final-state interactions in unitarized description (replacement of a sum of Breit-Wigner terms by analytical meson-meson amplitude)

Menu du Jour

I. Summary of $B^{\pm} \to (\pi\pi)_S K^{\pm}$ results by A. Furman *et al*. Phys. Lett. B622, 207 (2005)

II. Extension to $\pi\pi$ and $\bar{K}K$ interactions in *P*-wave

III. Current improvements

I. Three-Body Decay Reactions in S-Wave

Examples of *quasi two-body* reactions:

$B^{\pm} \to f_0(980)K^{\pm}$ *with subsequent* $f_0(980) \to (\pi^+\pi^-)_S$ *or* $f_0(980) \to (K^+K^-)_S$,

where $(\pi^+\pi^-)_S$ *and* $(K^+K^-)_S$ *mean* $\pi^+\pi^-$ *and* $K\bar{K}$ *pairs*

in S-wave I=0 state.

The effective mass range is: $2m_\pi < m_{\pi\pi} < 1.2$ GeV

Decay Amplitudes for $B \to (\pi\pi)_S K$ and $B \to (KK)_S K$ reactions

- Weak decay amplitudes $b \to u\bar{u}s$ and $b \to s\bar{s}s$ in QCD factorization approximation built from NLL Wilson coefficients following Beneke *et al.* Nucl. Phys. B606, 245 (2001) and de Groot *et al*., Phys. Rev. D **68**, 113005 (2003)

- Hard scattering with spectator quark and annihilation topologies not included (*later…*)

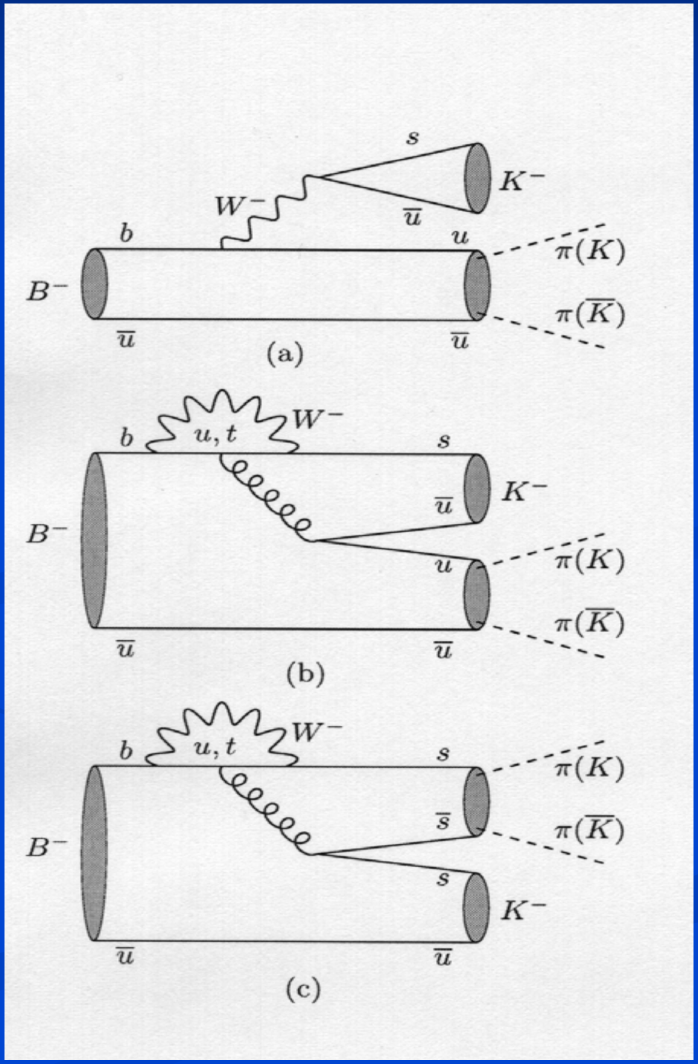
Quark-Line Topologies for $B \to f_0(980)K$

❶ Example:

$$\begin{cases} B^- \to (\pi\pi)_S K^- \\ B^- \to (KK)_S K^- \end{cases}$$

$$\left.\begin{array}{l} (\pi\pi)_S : \pi^+\pi^- \text{ or } \pi^0\pi^0 \\ (KK)_S : K^+K^- \text{ or } K^0\bar{K}^0 \end{array}\right\} \text{Isospin zero S-wave}$$

❷ For B^0 decays no tree diagram (a),
only penguin diagrams similar to ones in (b) or (c)



Formation of Meson Pairs

The $u\bar{u}$ or $s\bar{s}$ transition into $\pi\pi$ or $K\bar{K}$ described by *4 scalar form factors* by Meißner and Oller, Nucl. Phys. **A679**, 671 (2001):

$$n\bar{n} = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \qquad \begin{cases} \Gamma_1^n(m) = \langle 0|n\bar{n}|\pi\pi\rangle / \left(\sqrt{2}B_0\right) \\ \Gamma_2^n(m) = \langle 0|n\bar{n}|K\bar{K}\rangle / \left(\sqrt{2}B_0\right) \\ \Gamma_1^s(m) = \langle 0|s\bar{s}|\pi\pi\rangle / \left(\sqrt{2}B_0\right) \\ \Gamma_2^s(m) = \langle 0|s\bar{s}|K\bar{K}\rangle / \left(\sqrt{2}B_0\right) \end{cases}$$

$$B_0 = -\langle 0|\bar{q}q|0\rangle / f_\pi^2 \qquad B_0 = m_\pi^2 / (2\hat{m}),\ \hat{m} = \frac{1}{2}\left(m_u + m_d\right) = 5\ \text{MeV}$$

$f_\pi = 92.4$ MeV

Complete Amplitude for $B \to (\pi\pi)_S K$

$$\left\langle \left(\pi^+\pi^-\right)_S K^- \left| H_{\text{eff}} \right| B^- \right\rangle = \frac{G_F}{\sqrt{2}}\sqrt{\frac{2}{3}} \left\{ \chi \left\{ f_k \left(M_B^2 - m_{\pi\pi}^2 \right) F_0^{B\to(\pi\pi)_s} \left(M_K^2 \right) \times \right.\right.$$

$$\times V_{ub}V_{us}^* \left[a_1 + a_4^u - a_4^c + \left(a_6^c - a_6^u \right) r \right] + V_{tb}V_{ts}^* \left(a_6^c r - a_4^c \right) + C\left(m_{\pi\pi}\right) \Big\} \Gamma_1^{n*}\left(m_{\pi\pi}\right)$$

$$+ \left\{ \left[2\sqrt{2} B_0 / \left(m_b - m_s\right) \right] \left(M_B^2 - M_K^2 \right) F_0^{B\to K} \left(m_{\pi\pi}^2 \right) \times \right.$$

$$\left.\left. \times \left[V_{ub}V_{us}^* \left(a_6^c - a_6^u \right) + V_{tb}V_{ts}^* a_6^c \right] + \chi C\left(m_K\right) \right\} \Gamma_1^{s*}\left(m_{\pi\pi}\right) \right\}$$

Normalization constant χ fitted to branching ratio $B^\pm \to f_0(980)K^\pm$ but can also be estimated from:

$$\chi \approx \frac{g_{f_0\pi\pi}}{\left[m_{f_0} \Gamma_{\text{tot}}\left(f_0\right) \left| \Gamma_1^n\left(m_{f_0}\right) \right| \right]} \cong 30\ \text{GeV}^{-1}$$

Charming Penguins $C(m)$

If $C(m)=0,\ Br\left[B^{\pm}\to f_0(980)K^{\pm},\ f_0\left(980\right)\to\pi^+\pi^-\right]$

is *too* small by a factor of **4** : cancellation due to $a_4^c \approx a_6^c$ and $r \approx 1$

➲ Long distance contribution, considered by N. de Groot *et al.* to improve their fit to hadronic charmless strange and non-strange two-body decays

➲ B-decay data: **CHARMING PENGUINS: ENHANCED CHARM QUARK LOOPS**
M. Ciuchini, E. Franco, G. Martinelli, M. Pierini and L. Silvestrini, Phys. Lett. **B515**, 33 (2001)

They could correspond to $B^-\to D_s^- D^0 \xrightarrow[\left[c\bar{c}\ \text{annihilation}\right]]{} f_0(980)K^-$

Parametrized as:

$$C(m) = -\left(M_B^2 - m^2\right) f_\pi F_\pi^{B\to\pi}(0)\left(V_{ub}V_{us}^* P_1^{GIM} + V_{tb}V_{ts}^* P_1\right)$$

$m = m_{\pi\pi}$ or m_K, $F^{B\to\pi}(0)$: $B\to\pi$ transition form factors

P_1^{GIM}, P_1 are complex parameters determined by de Groot et al. and also in by M. Ciuchini *et al.* to fit some charmless two-body B-decay data.

Final-State Interactions

There are two coupled channels in the *S*-wave (*I* = *0*) state: $\pi\pi$ and $K\bar{K}$

$T(\pi\pi\to\pi\pi)$ $\quad T(\pi\pi\to K\bar{K})$

$T(K\bar{K}\to\pi\pi)$ $\quad T(K\bar{K}\to K\bar{K})$

The 4 scalar form factors $\Gamma_i^{n,s}(m)$ are incorporated in the unitarity-constraint formulae:

$$\Gamma_i^{n,s}(m) = R_i^{n,s}(m) + \sum_{j=1}^{2} \left\langle k_i \right| R_j^{n,s}(m)\, G_j(m)\, T_{ij}(m) \left| k_j \right\rangle$$

$$\begin{cases} |k_i\rangle, |k_j\rangle : \text{two meson wave function in momentum space} \\ i, j = 1\ (\pi\pi),\ 2\ \left(K\bar{K}\right); \text{T : two-body scattering matrix of } \left\{\pi\pi,\ \text{K}\bar{\text{K}}\right\} \text{ coupled channel} \end{cases}$$

- Model of R. Kamiński, L. Leśniak and B. Loiseau, 1997 and 1999
  $G_i\,(m)$: free Green's function.

Production functions $R_{i,j}^{n,s}(m)$ initial formation of meson pair prior to scattering, derived by Meißner and Oller in one-loop chiral perturbation theory:

$$\left.\begin{cases} R_1^n(m) = 0.566 + 0.414m^2 \\ R_2^n(m) = -0.322 + 0.527m^2 \\ R_1^s(m) = -0.036 + 0.353m^2 \\ R_2^s(m) = 0.071 + 0.338m^2 \end{cases}\right\} m \text{ in GeV}$$

Using on shell contribution for $\Gamma_i^{n,s}(m) \Rightarrow$

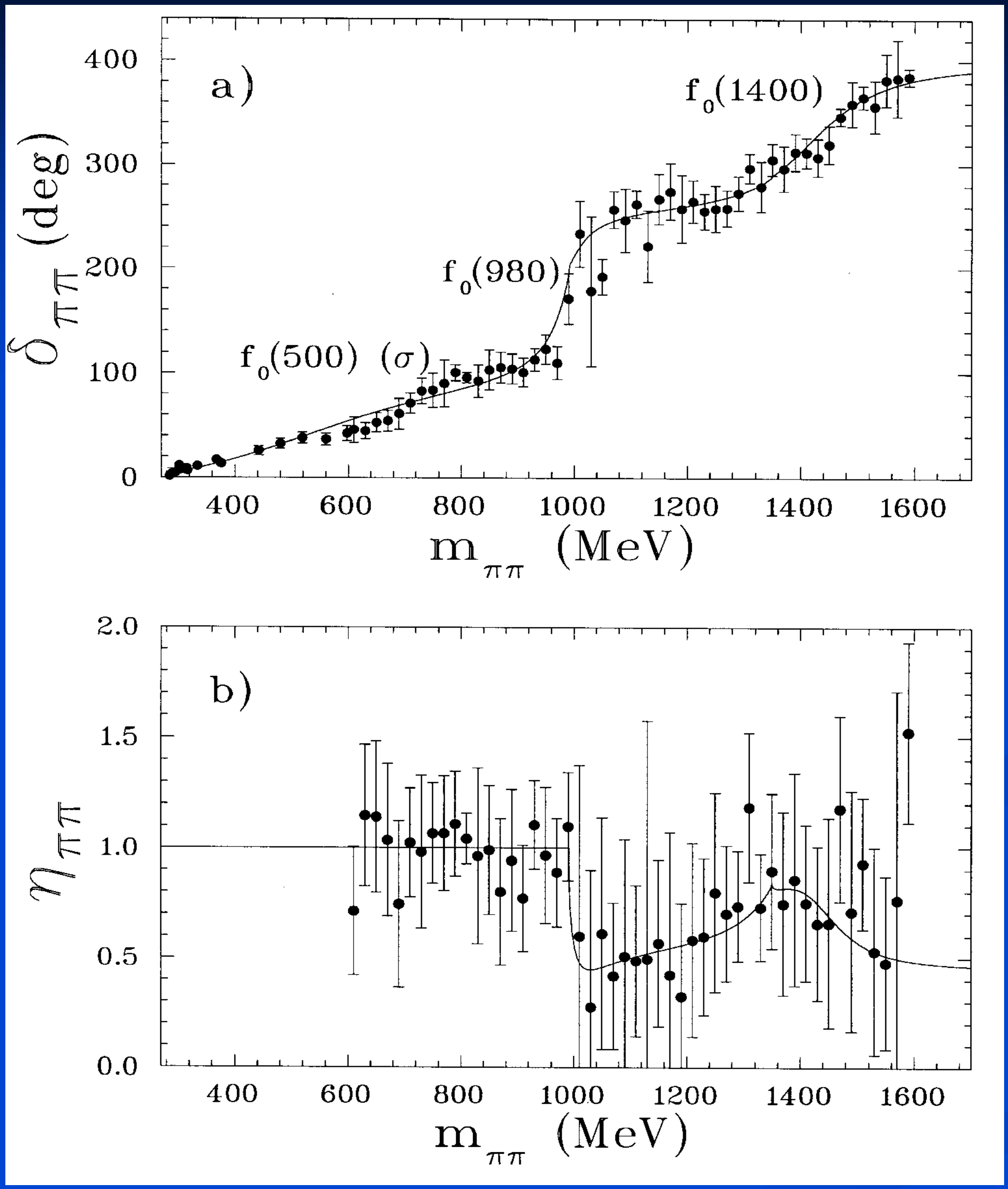
$$\Gamma_1^{n,s*}(m) = \frac{1}{2}\left[R_1^{n,s}(m)\left(1+\eta(m)\, e^{2i\delta_{\pi\pi}(m)}\right) - i\, R_2^{n,s}(m)\sqrt{\frac{k_2}{k_1}}\sqrt{1-\eta^2(m)}\; e^{i\left(\delta_{\pi\pi}(m)+\delta_{K\bar{K}}(m)\right)} \right]$$

$$\Gamma_2^{n,s*}(m) = \Gamma_1^{n,s*}(m, 1 \leftrightarrow 2)$$

Below $K\bar{K}$ threshold: $\eta(m) = 1:\ \Gamma_1^{n,s*}(m) = R_1^{n,s}(m)\ \cos\delta_{\pi\pi}(m)\ e^{i\delta_{\pi\pi}(m)}$

* if $\delta_{\pi\pi}$ close to 180° $\Rightarrow$ maximum for $\left|\Gamma_1^{n,s}\right| \leftrightarrow$ case for $f_0(980)$

* $\Gamma_1^{n,s} = 0$ for $\delta_{\pi\pi} = \pi/2$



Energy dependence of $I=0$ S-wave: a) $\pi\pi$ phase shifts and b) $\pi\pi$ inelasticity

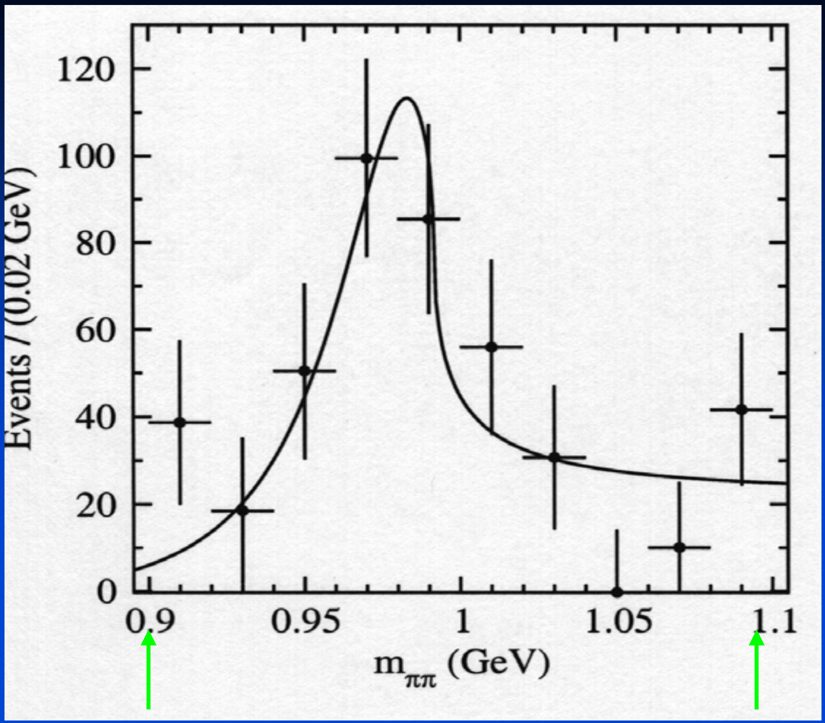
Results for $B \to f_0(980)K$

Average branching fractions in units of 10^{-6}

Model I: $C(m)$ of de Groot *et al.*
Model II: $C(m)$ of Ciuchini *et al.*

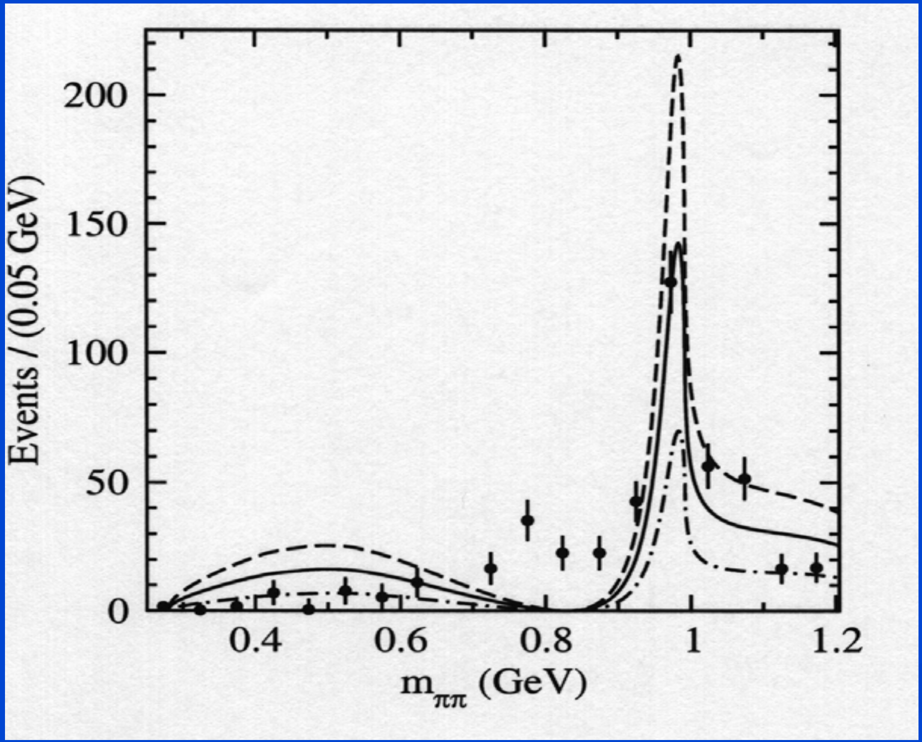
Model errors from uncertainty on $C(m)$; $A(t) = S \sin(\Delta m t) + A \cos(\Delta m t)$

$a)\ B^{\pm} \rightarrow \pi^{+}\pi^{-}K^{\pm}$ decays



Comparison with BaBar (2005) **Model I**

$\Rightarrow \chi = 35\ \text{GeV}^{-1}$ [integration from 0.9 to 1.1 GeV]



Comparison with Belle (2005)

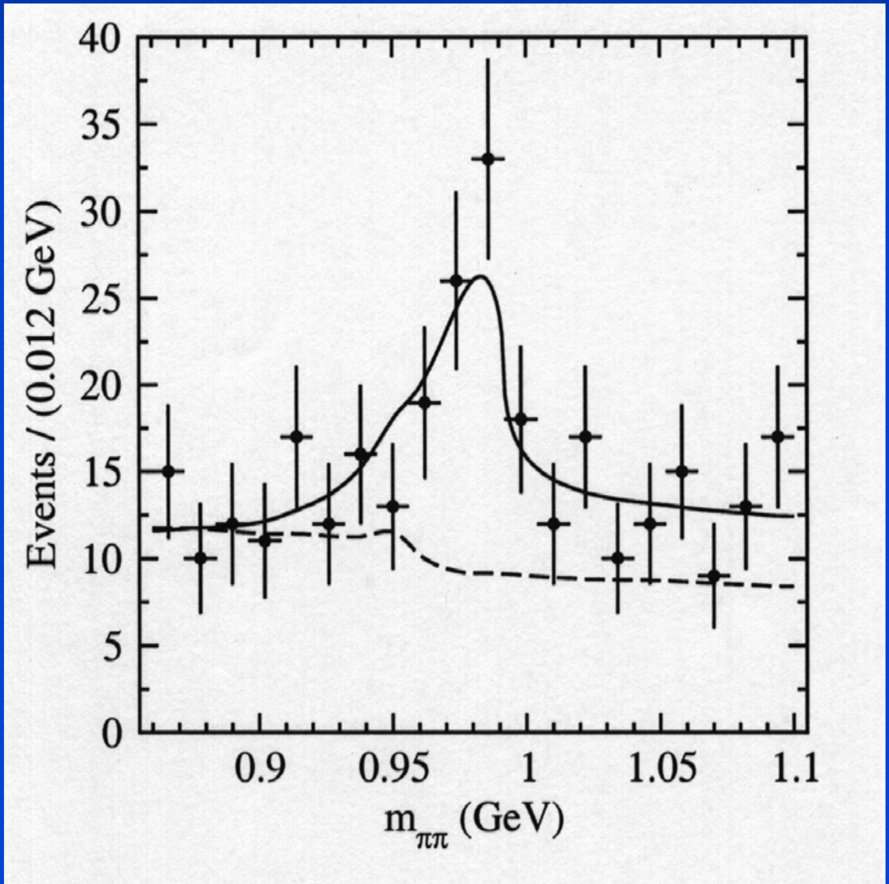
$B^{+} \rightarrow \pi^{+}\pi^{-}K^{+}$: dashed line

$B^{-} \rightarrow \pi^{+}\pi^{-}K^{-}$: dotted line

Average : solid

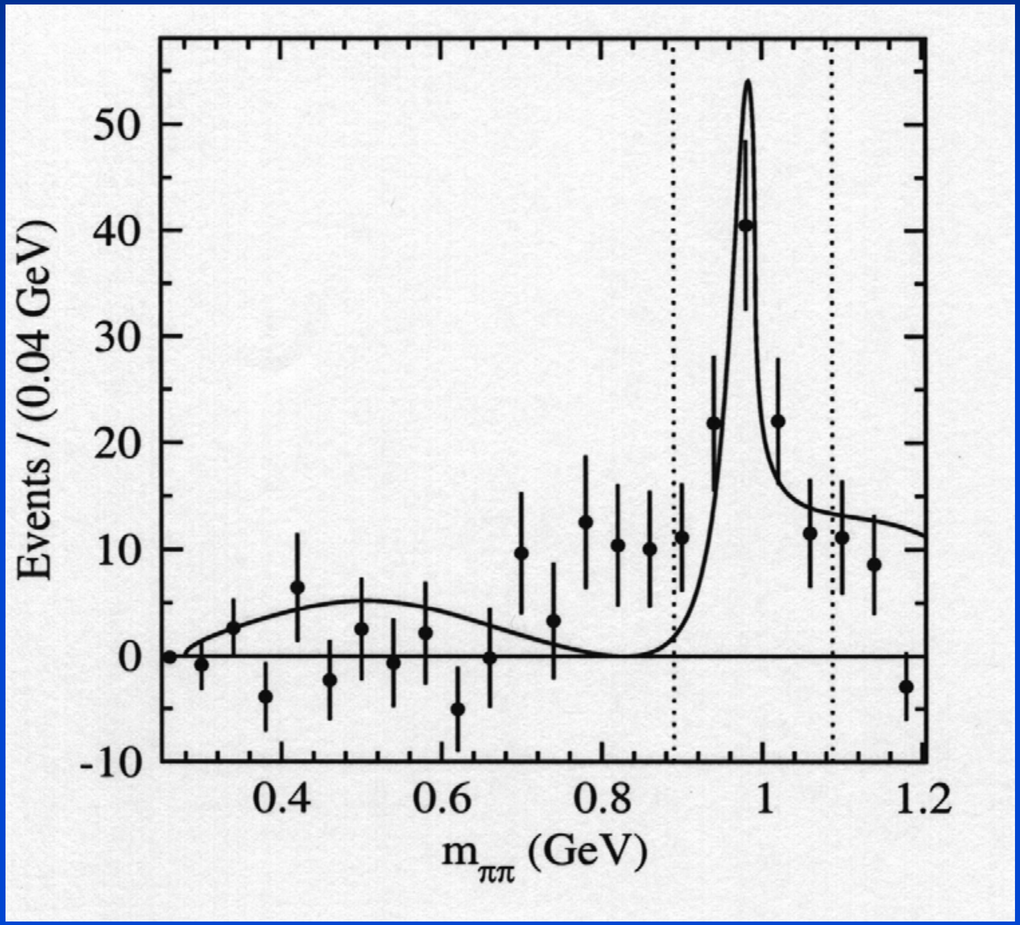
b) $B^0 \to \pi^+\pi^- K^0$ decays

Comparison with BaBar (2005) **Model I**



➲ If *C(m)=0*, prediction drops by a factor of 18: near cancellation of the two-penguin diagrams + absence of tree diagram

Comparison with the $\pi^+\pi^-$ spectrum of Belle (2004) for $B^0 \to \pi^+\pi^- K_S^0$



Model I

- Note non-zero contribution at $f_0(600)$ in *S*-wave.
- Presence of ρ (770) ($\delta_{\pi\pi} = \pi/2$)

II. Three-Body Decay Reactions in *P*-Wave

- Clear presence of ρ(770) in effective mass $m_{\pi\pi}$ distribution
- In previous work *P*-wave interactions not included
- Experimental branching ratio for $B^{\pm} \to \rho(770)^0 K^{\pm}$:

  $3.89 \pm 0.47 \pm 0.29^{+0.32}_{-0.29} \times 10^{-6}$ (Belle 2005)

  $5.08 \pm 0.78 \pm 0.39^{+0.22}_{-0.66} \times 10^{-6}$ (BaBar 2005)

  Large $A_{CP} = +30 \pm 11 \pm 3.0^{+11}_{-4}$ (Belle)

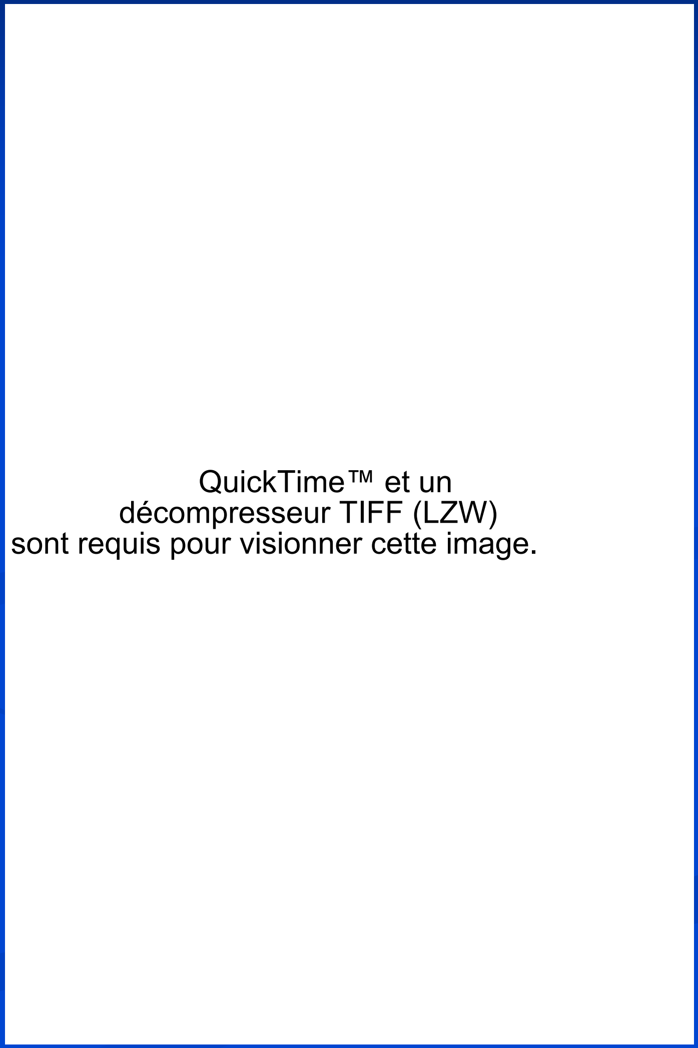
  $A_{CP} = +34 \pm 13 \pm 6.0^{+15}_{-20}$ (BaBar)
- QCDF predictions for $Br[B^{-} \to \rho^0 K^{-}]$ are *lower*:

  1.54×10^{-6}, Leitner *et al*., J. Phys. G31, 199 (2005)

  2.6×10^{-6}, Beneke and Neubert, Nucl. Phys. B675, 333 (2003)

Quark-Line Topologies for $B^{\pm} \to \rho^0 K^{\pm}$, $\rho^0 \to (\pi\pi)_P$

- One additional tree diagram



The amplitude is

$$\langle (\pi^+\pi^-)_P K^- | H | B^- \rangle = 2 A_{VP}(m_{\pi\pi}) \Gamma_{\rho\pi\pi}(m_{\pi\pi}) |p_\pi| |p_K| \cos\theta$$

with

$$A_{VP}(m_{\pi\pi}) = G_F m_\rho [f_K A_0^{B\to\rho}(m_K) U + f_\rho F_0^{B\to K}(m_\rho) W + C(m_{\pi\pi})],$$

where U and W are functions of CKM elements and weak decay effective coefficients $a_i(\mu)$ in QCD factorization, $C(m)$ possible charming penguin contributions.

For the $\rho \to \pi\pi$ vertex function a Breit-Wigner function is used

$$\Gamma_{\rho\pi\pi}(m_{\pi\pi}) = \frac{g_{\rho\pi\pi}}{m_{\pi\pi}^2 - m_\rho^2 + i\Gamma_\rho m_\rho}$$

where $m_\rho = 775.8$ MeV, $\Gamma_\rho = 150.3$ MeV, $g_\rho = 3m_\rho^2\,\Gamma_\rho/(2p_\pi^2) = 6$

Adding the *S*- wave contributions:

$$M = a_S + a_P \left| p_\pi \right| \left| p_K \right| \cos\theta$$

a_S : *S*-wave contribution

$$a_P = 2A_{VP}(m_{\pi\pi})\Gamma_{\rho\pi\pi}(m_{\pi\pi})$$

III. Current Improvements

- Introduction of annihilation diagrams
- Taking into account hard-scattering with spectator quark
- Study of importance of charming penguins